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ABSTRACT

This book introduces students to the types of problems and processes used throughout the "Reshaping Mathematics for Understanding" series. The problems in this unit deepen students' understanding of mathematics by encouraging them to clarify concepts and challenge their own assumptions. Additionally, by providing opportunities to give and follow directions, create representations, and explain their thinking, the problems encourage students to recognize and accept multiple strategies and to be flexible in their thinking. This book encourages students to ask questions, test ideas, and offer alternative points of view. These behaviors allow students to practice the skills they need to communicate effectively in a mathematics classroom and encourage them to share the responsibility for making the class successful. Five strands include: (1) Communication; (2) Problem Solving; (3) Representation; (4) Visual/Spatial Thinking; and (5) Motion. (KHR)

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Reshaping Mathematics for Understanding

Getting Started

ED 482 952

Hannah Slovin
Linda Venenciano
Melanie Ishihara
Cynthia Beppu

U.S. DEPARTMENT OF EDUCATION
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About the series . . .

Reshaping Mathematics for Understanding

Reshaping Mathematics for Understanding is a series of fourteen units suitable for sixth- through eighth-grade students that addresses important topics in middle-grades mathematics, including geometry, measurement, proportional reasoning, rational numbers, probability and statistics, and algebra. The entire series is designed to help students learn to think mathematically. It focuses on developing students' understanding of mathematical concepts and on their ability to draw connections among these concepts. It may serve either as the primary resource in the mathematics curriculum or as a complement to other material. The titles for the full series are listed below.

- Getting Started
- Motion Geometry
- Measurement
- Polygons
- Dilations
- Fractions
- Decimals
- Ratio and Proportion
- Area of Polygons
- Solids
- Probability and Statistics
- Integers
- Algebra Patterns and Relationships
- Number Theory

Access for Every Student

The problems and tasks in the *Reshaping Mathematics for Understanding* series are designed to enable every student to approach mathematics through familiar contexts. Building problem situations on students' past experience makes the study of mathematics more accessible and allows them to expand their thinking. Similarly, to promote genuine engagement, many problems have more than one solution path, giving students opportunities to choose the strategies they prefer using to solve problems. Additionally, to give students opportunities to interact with each concept at varying levels of abstractness and generality, the lessons present new ideas over several days. This design feature also allows students to learn at different rates.

Learning with Understanding

Students can learn only what they truly understand. To learn, they must understand the concepts that underlie operations; they must make connections among processes and concepts; and they must know when and how to apply concepts and operations to problems. Understanding mathematics means doing more than learning how to perform calculations to get correct answers. The topics in this series, developed through problems and lab explorations, encourage students to deepen their conceptual understanding through the practice of reasoning and problem solving.

Role of Visual/Spatial Thinking and Reasoning

Mathematics can and should be a lively course of study for students. It should engage them in active inquiry and give them many opportunities to explore problems whose solutions add to their understanding of the world. For many students, however, mathematics involves merely manipulating numbers. With this limited view, they do not learn how to use multiple contexts, tools, and strategies to solve problems or how to integrate mathematical concepts into broader contexts. Transformational (motion) geometry, which plays an important role in the sequence of the concepts in this series, emphasizes the use of visual contexts and spatial thinking. Lesson discussions further encourage students to use their understanding of spatial relations to make connections among concepts. The Motion Geometry and Dilations units, in particular, give students valuable experience with transformations and enhance the study of many related topics.

Unit Design

Each unit in the *Reshaping Mathematics for Understanding* series can be used individually, or units can be used in clusters. Throughout the series, references among units direct teachers and students to tasks that will help them connect their understanding of new concepts to related experiences.

The lessons in each unit are uniquely designed to enable students to progress through a sequence of tasks that maximize learning with understanding. Rather than present a topic in its entirety in one day, the lessons develop concepts over time. Most concepts begin with an open-ended problem that draws on students' previous experience and intuition and allows for multiple responses. The variety of solutions students propose to such a problem helps the teacher assess what background knowledge students bring to the topic. The problems and tasks that follow develop the concept through a sequence of approaches that provide several direct examples of the concept, raise questions for clarification, offer alternative viewpoints, and prompt students to summarize ideas.

The Lessons

There are two forms of lessons in each unit, problem sets and in-class labs. Problem sets consisting of three to five problems are designed to cover several concepts related to the unit topic. Problem sets should be assigned for homework and discussed in class the next day. When students find a problem too difficult to solve on their own, they should be instructed to write questions to help them solve it, and to ask the questions in the discussion. Students work on the labs in small groups in class and debrief afterward, giving them an opportunity to work collaboratively and to concentrate on one strand.

The class discussions are essential in helping students build conceptual understanding. Sharing their solutions and questions allows students to reflect on their thinking and to consider input from others. Both the teacher and students share the responsibility for making discussions productive. As they solve the problems and complete the lab tasks, students explain their thinking, offer alternative responses, and ask questions. Teachers facilitate, asking strategic questions to focus students' thinking on critical ideas. In guiding the discussion, teachers should ensure engagement in the learning process by encouraging students to monitor their learning and by providing a safe, open learning environment in which to share, discuss, and address misconceptions.

Assessment

Except for the Getting Started unit, designed to orient students to the series, all units include suggested assessment items. To check periodically that students understand and that they are participating in the class discussions, teachers can also create "instant" quizzes to give the day after a problem set discussion or lab debriefing. These quizzes should be unannounced and should take between five and ten minutes at the start of class. They have three purposes:

to emphasize the importance of student discussions;

to emphasize the value of information shared by students;

to highlight the importance of asking questions to clarify understanding.

In creating an instant quiz, teachers should focus on students' ideas from the discussion and write two or three short-answer questions as follows:

- a question that is content-based or refers to an understanding that has been established.

  Example: What did we decide it meant to measure the perimeter of a polygon?

- a question that refers to someone's method for solving a problem or an alternative point of view.

  Example: What method did Leslie use to find the area of the trapezoid?

- a question that highlights an undefined assumption or a statement that needed clarification.

  Example: What did Jose mean by *proportional*?

Materials

A list of the materials needed for each lesson appears at the front of every unit. Although students need no special materials or equipment to complete the homework problems, it would be helpful to have them available during discussions. Encouraging students to use a broad range of tools to explore and express their thinking promotes greater understanding.

About the unit . . .

The Getting Started unit introduces students to the types of problems and processes used throughout the *Reshaping Mathematics for Understanding* series. The unit may be introduced at the beginning of the school year or whenever it is appropriate to teach strategies that use problem solving to develop mathematics concepts. As students solve the problems in this unit and discuss their solutions, they practice behaviors that deepen their understanding and contribute to a positive learning environment.

Deepening Understanding

The problems in the Getting Started unit deepen students' understanding of mathematics by encouraging them to clarify concepts and challenge their own assumptions. Additionally, by providing opportunities to give and follow directions, create representations, and explain their thinking, the problems in this unit encourage students to recognize and accept multiple strategies and to be flexible in their thinking.

Creating a Positive Learning Environment

The Getting Started unit encourages students to ask questions, test ideas, and offer alternative points of view. These behaviors allow students to practice the skills they need for communicating effectively in a mathematics classroom and encourage them to share the responsibility for making the class successful. At the same time, the problems ask students to explain their thinking, ask productive questions, and challenge answers without being negative. Engaging in mathematics tasks that are specifically designed for students to practice desired behaviors and to establish social norms is the most effective means for building a positive classroom environment.

The Strands in Getting Started

Communication: To communicate effectively, students must explain, describe, clarify, interpret, and question. The problems in this unit are designed to generate active inquiry. Problems that are open to interpretation motivate students to explain their points of view. Encountering problems that they may solve in more than one way or that have more than one correct answer encourages them to exchange ideas and build understanding. Having to describe and explain their processes makes them more aware of their own thinking and gives them more control over when to apply problem-solving strategies. Communication also helps students learn other ways to solve problems. They often ask each other, "How did you know to use that method?" The thinking that underlies the strategy is often a mystery to them. A focused inquiry helps them to be more conscious of the thought processes they use to solve problems.

Problem Solving: While the entire unit takes a problem-solving approach, this strand introduces key processes essential in helping students develop the strategies they need to become effective problem solvers. First, they must understand that solving problems requires

them to make certain assumptions about the problem conditions. The problems in this strand lead them to recognize that their assumptions stem from their previous knowledge and experience. Becoming aware of their assumptions leads them to recognize a wider range of possible contexts for problems. Second, to solve these problems, students must use organized strategies. Problems that require them to identify, describe, and extend patterns give them valuable strategic experience in recognizing mathematical relationships. Lastly, to teach them that they must learn to test their solutions independently, many problems in this strand ask them to explain and justify their solutions.

Representation: The ability to represent and interpret representations of problem situations enhances students' problem-solving abilities. Representation helps students mathematically communicate how they understood the problem. In the Getting Started unit, students use diagrams, tables, and symbols to create and interpret mathematical situations.

Visual/Spatial Thinking: Visual/spatial thinking helps all students understand and solve problems. Students who have not attained fluency with numbers may see patterns and relationships more easily when they are represented geometrically. Exercising visual/spatial skills also builds students' conceptual understanding and prepares them for more abstract problem-solving tasks. The entire *RMU* series offers many opportunities for students to use visual/spatial thinking.

Introduction to Motion: Transformational-geometry activities are important in middle-grades mathematics. They provide a context for students to observe figures, reason about relationships among figures, and make connections to important middle-grades topics such as proportion. The problems in the Introduction to Motion strand of this unit ask students to build informal proofs about figures and their properties, a practice that develops the understanding they will need in high school mathematics courses.

Concept Development

The numbers in this matrix refer to the problem numbers in each lesson.

STRAND and Focus	Lab A	Lab B	PS 1	PS 2	PS 3	PS 4	PS 5	PS 6
Communication								
Giving/Following Directions	1–4	Tie a Bow						3
Description			2					
Problem Solving								
Making Assumptions			1		1			
Numerical Patterns					3		1	3
Problem Conditions				1				
Testing Solutions							1	
Representation								
Interpreting Diagrams				3		2		
Representing Motion								3
Visual/Spatial Thinking								
Networks				4		1		
Reconfiguration					2		2,3	1,2
Visual Patterns			2	1	4	2		2
Testing Perceptions			3	2,3		3		
Introduction To Motion								
Using Motion in Problem Solving				2	4	3		
Congruence								2

x

Materials

Special materials for Getting Started

Lab A	Drawing Activity Cards (see blackline masters at the end of the unit), 1 per student
Lab B	Shoe with ties or laces, 1 per student
Problem Set 1	Geometric compass, 1 per group Straight-edge*, 1 per student Square tiles, 20 per group
Problem Set 2	Straight-edge*, 1 per student Tracing paper
Problem Set 3	Counters, 20 per group Toothpicks, 20 per group Tracing paper
Problem Set 4	Geometric compass, 1 per group Cubes such as centimeter cubes or connecting cubes, 70 per group Straight-edge*, 1 per group Tracing paper
Problem Set 5	Toothpicks, 20 per group
Problem Set 6	Dimes and pennies, 3 of each per group Square tiles, 70 per group Toothpicks, 20 per group

*Straight-edges without unit markings are preferable to rulers. Blank index cards could also be used.

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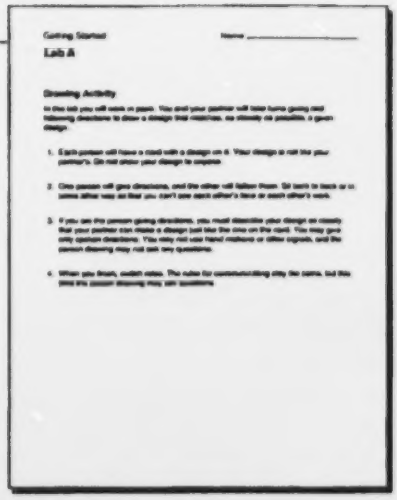
Lab A

Giving Clear Directions



Lab A, Nos. 1–4

Strand: Communication

Focus: Giving/Following directions

Task: Rehearsing roles as givers and receivers of information

PROBLEM

Drawing Activity

In this lab you will work in pairs. You and your partner will take turns giving and following directions to draw a design that matches, as closely as possible, a given design.

1. Each person will have a card with a design on it. Your design is not like your partner's. Do not show your design to anyone.
2. One person will give directions, and the other will follow them. Sit back to back or in some other way so that you can't see each other's face or each other's work.
3. If you are the person giving directions, you must describe your design so clearly that your partner can make a design just like the one on the card. You may give only spoken directions. You may not use hand motions or other signals, and the person drawing may not ask any questions.
4. When you finish, switch roles. The rules for communicating stay the same, but this time the person drawing may ask questions.

Teacher's Insight

Students are important sources of information in mathematics class discussions. In productive discussions, many students participate, proposing ideas, questioning ideas, and assessing other viewpoints. In this activity, students practice giving and receiving information needed to complete a task. It is crucial that they be as clear as possible in giving directions and that they listen carefully to their partners. Even students who have worked in small groups may have little experience in collaborating to complete a task.

Have students read all the directions, making sure they understand the procedure. In the first round, students who are drawing may not ask any questions. In the second round they may. This change helps them grasp how helpful asking questions can be when someone is explaining a complex idea.

The debriefing gives students a chance to assess the effectiveness of their communication. Remind them to concentrate on what worked well and what they could improve. Help them assess their drawings to see how well they captured the details of the design.

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ANSWER

Drawings from the second round should be more accurate than those from the first round.

★HOMEWORK

Write directions for tying a bow.

Discussion

How did the drawings turn out?

Look carefully at the details (position, shape, size, and orientation).

How did you decide if a drawing was good?

To direction-givers: What was hardest to communicate? What was easy?

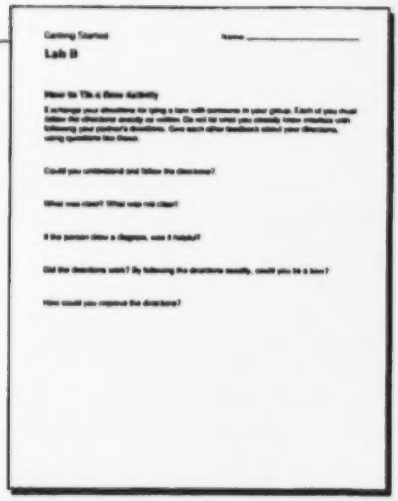
To drawers: What was hardest to picture in your mind? What was easy?

How did you handle the challenges?

How did not being able to ask and answer questions make this task more challenging?

How does asking questions help you understand?

Following Directions: How to Tie a Bow



Lab B

Strand: Communication

Focus: Giving/Following directions

Task: Identifying and describing the steps in doing a routine task

PROBLEM

How to Tie a Bow Activity

Exchange your directions for tying a bow with someone in your group. Each of you must *follow the directions exactly as written.* Do not let what you already know interfere with following your partner's directions.

Give each other feedback about your directions, using questions like these:

Could you understand and follow the directions?

What was clear? What was not clear?

If the person drew a diagram, was it helpful?

Did the directions work? By following the directions exactly, could you tie a bow?

How could the directions be improved?

Teacher's Insight

Many mathematics problems call for writing about the process used to solve a problem. Students must communicate their understanding meaningfully and accurately. They must analyze a strategy for completing a task and convey that method to someone else. This activity gives students the chance to practice becoming conscious of how they perform a familiar action and to write about it. Using diagrams and written explanations helps them to communicate effectively.

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ANSWER

Students' directions should be detailed and specific. Students may include step-by-step diagrams with their directions.

★HOMEWORK

Problem Set 1

Discussion

Before starting the discussion, have students give written feedback to their partners using these prompts:

"I wasn't sure about . . ."

"Something you put in your directions that helped was . . ."

What were some ways that this person made the directions easier to follow?

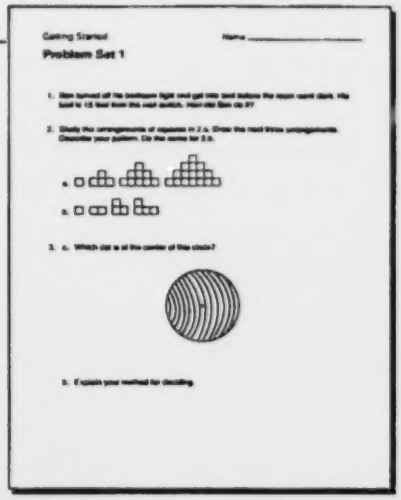
What do you think this activity has to do with math class?

What are your opinions about this homework assignment? Was it easy or difficult? Why?

What made this activity easy? What made this activity difficult? Explain.

Problem Set 1

Explaining Your Answer



Problem Set 1, No. 1

Strand: Problem Solving

Focus: Making assumptions

Task: Using flexibility and creativity in problem solving

PROBLEM

1. Ben turned off his bedroom light and got into bed before the room went dark. His bed is 15 feet from the wall switch. How did Ben do it?

ANSWER

1. Ask the class to share a variety of answers. One answer might be that Ben went to bed before the sun went down so the sunlight kept his room from being dark even after he turned the light off.

Teacher's Insight

Brainteasers give students opportunities to be creative and open-minded in deciding on the reasonableness of answers and to become aware of how their own assumptions influence their thinking.

Discussion

What made this problem difficult?

If students talk about needing to have a "math" answer, ask what a "math" answer would be. Why do you think you need it?

Are all solutions sensible? Why or why not?

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Problem Set 1, No. 2

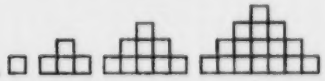
Strand: Communication and Visual/Spatial Thinking

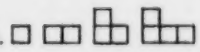
Focus: Description and visual patterns

Task: Describing and projecting visual patterns

PROBLEM

2. Study the arrangements of squares in 2.a. Draw the next three arrangements. Describe your pattern. Do the same for 2.b.

a. 

b. 

ANSWER

2. a. Add a row at the bottom that has two more blocks than the previous row.

   b. Alternate the addition of a block to the right and a block to the top.

Teacher's Insight

In this problem students describe mathematical patterns. They sometimes suggest schemes for adding arrangements that do not follow the pattern. The discussion should address what makes a series of arrangements form a pattern. After discussing the patterns, have students discuss "rules" for finding larger arrangements.

Discussion

Have students share their descriptions.

How can you assess if a description makes sense?

Did any particular portion of the pattern give you a clue to what the pattern was?

How could you test to see if your drawings were correct?

Would you be able to skip an arrangement and still come up with the next one in the series? For example, what would the ninth or tenth arrangement in the series look like?

Was one pattern more difficult to project or describe than the other? If yes, why?

What is a pattern?

Problem Set 1, No. 3

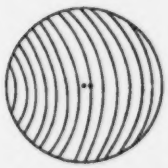
Strand: Visual/Spatial Thinking

Focus: Testing perceptions

Task: Reasoning mathematically to justify an answer

PROBLEM

3. a. Which dot is at the center of this circle?



b. Explain your method for deciding.

ANSWER

3. a. The left dot is at the center of the circle.

b. Ask the class to share a variety of explanations on how they decided on their answer. One explanation might be to fold the circle in half, such that both dots aren't in the fold, and decide that the dot that is in the fold is the center of the circle.

★HOMEWORK

Problem Set 2

Teacher's Insight

This is an optical illusion problem designed to help students form the habit of testing and justifying their perceptions. Students sometimes devise complicated methods to solve this problem. If they talk about measuring, ask how they can verify the accuracy of their measurement. The simplest mathematical approach is to fold the circle in half (but not along the diameter on which both dots lie) and to identify the dot that lies in the fold.

Discussion

How did you decide where the center was?

What method did you use?

How could you test your answer?

Should everyone get the same results no matter what method they used?

When students talk about folding the circle in half to find the center, ask what it means to fold something in half.

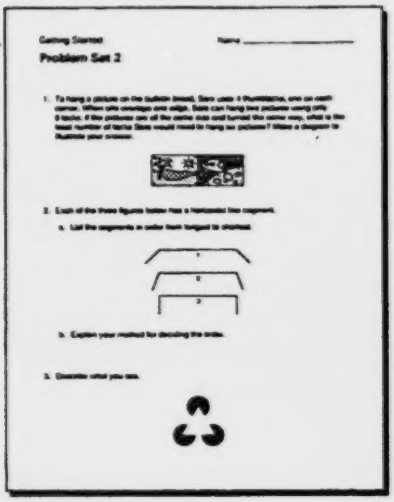
If students measured, ask where they measured. How did measuring help you decide?

Is it important to test what you think you observe? Why or why not?

Why are some methods more convincing than others?

Problem Set 2

Using Diagrams



Problem Set 2, No. 1

Strand: Problem Solving and Visual/Spatial Thinking

Focus: Problem conditions and visual patterns

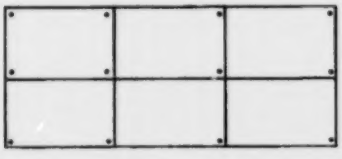
Task: Using a diagram to solve a problem with multiple conditions

PROBLEM

1. To hang a picture on the bulletin board, Sara uses 4 thumbtacks, one on each corner. When she overlaps one edge, Sara can hang two pictures using only 6 tacks. If the pictures are all the same size and turned the same way, what is the least number of tacks Sara would need to hang six pictures? Make a diagram to illustrate your answer.

ANSWER

1. The least number of tacks needed is 12.



Teacher's Insight

In this problem students must check their solutions against the conditions in the problem. They must understand they are to find the least number of tacks, and therefore this problem has only one correct answer.

Using a diagram makes it easier for students to check whether they have used the fewest tacks.

Discussion

Have students discuss their methods for finding the arrangement that fits the conditions.

What are the conditions of the problem? What does "turned the same way" mean?

How did you interpret what was meant by "the least number of tacks"?

Is there more than one possible arrangement that uses the least number of tacks?

Problem Set 2, No. 2

Strand: Visual/Spatial Thinking and Introduction to Motion

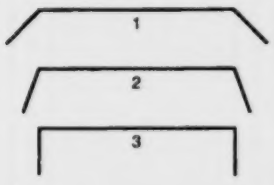
Focus: Testing perceptions and using motions in problem solving

Task: Using motions to compare line segments

PROBLEM

2. Each of the three figures below has a horizontal line segment.

   a. List the segments in order from longest to shortest.



   b. Explain your method for deciding the order.

ANSWER

2. The segments are congruent. Ask the class to share a variety of explanations on how they decided on their answer. One explanation might be to trace the first horizontal line segment then slide the tracing down to completely overlap each of the other two horizontal line segments. This test proves that they are all the same length.

Teacher's Insight

This is another optical illusion problem. Have students demonstrate how they tested their solutions. They either measure with a standard unit or use the edge of a paper to test the lengths of the segments. They sometimes disagree about the measurements they got. Guide students to focus their attention on figuring out the causes of their disagreements. Discuss how using different types of tests make the results convincing.

Discussion

What is a line segment?

How did you decide what the order was?

How did you test your order?

What are some other ways to test your solution?

Problem Set 2, No. 3

Strand: Representation and Visual/Spatial Thinking

Focus: Interpreting diagrams and testing perceptions

Task: Building vocabulary for describing and acknowledging different perceptions

PROBLEM

3. Describe what you see.

ANSWER

3. Ask the class to share a variety of explanations of how they decided on their answer. One explanation might be that they see three black circles with the same sized piece missing from each one.

Teacher's Insight

Here is another opportunity for students to use mathematical descriptions. Check that they are listening to and understanding each other's descriptions. Geometric shapes will not be as obvious to students as they are to you.

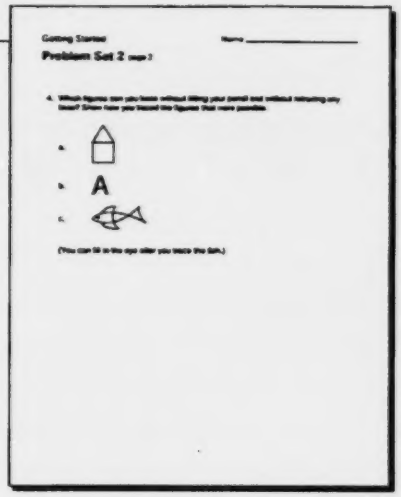
Discussion

What do you see?

Is there a difference between what somebody thinks is there and what is actually there? (Students sometimes talk about seeing a triangle, but no triangle is actually drawn.)

Why do you think this happens?

Why might different people see different things?



Problem Set 2, No. 4

Strand: Visual/Spatial Thinking

Focus: Networks

Task: Using flexibility to find multiple solutions in visual problem solving and to decide that no solution exists

PROBLEM

4. Which figures can you trace without lifting your pencil and without retracing any lines? Show how you traced the figures that were possible.

a.

b.

c.

(You can fill in the eye after you have traced the fish.)

ANSWER

4. The figures in 4.a. and 4.c. are possible but the figure in 4.b. is not. Ask students to share a variety of paths to solve 4.a. and 4.c.

★HOMEWORK

Problem Set 3

Teacher's Insight

Although this problem involves networks, it is not intended to teach students about networks at this time.

Students often have difficulty deciding that a problem cannot be solved, even after spending a long time trying to solve it. Students say they cannot trace figure 4.b. A few may try a trick using another piece of paper or a second pencil. But after their initial approval, some students say that the method is unacceptable because it would make it possible to trace any figure. This problem gives the class an opportunity to define what is a reasonable amount of time to work on one problem.

This problem can serve later as an introduction to discussing representation of motion.

Discussion

Describe your tracing.

Could you trace all the figures?

Did any figures have more than one possible starting point?

How did you show the path you followed on your homework paper?

How do you decide when a problem is impossible to solve?

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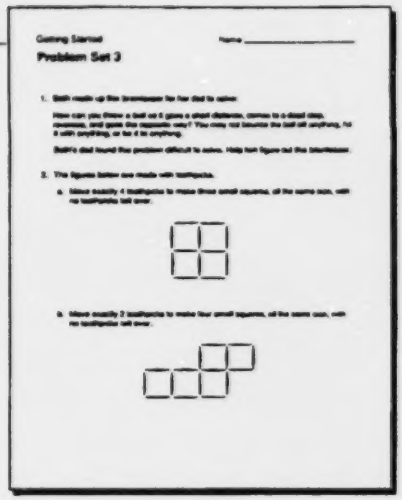
Demonstrating Your Solution



Problem Set 3, No.1

Strand: Problem Solving

Focus: Making assumptions

Task: Using flexibility and creativity in problem solving

PROBLEM

1. Beth made up this brainteaser for her dad to solve:

   How can you throw a ball so it goes a short distance, comes to a dead stop, reverses, and goes the opposite way? You may not bounce the ball off anything, hit it with anything, or tie it to anything.

   Beth's dad found this problem difficult to solve. Help him figure out this brainteaser.

ANSWER

1. Ask the class to share a variety of explanations on how they decided on their answer. One explanation might be that a person throws the ball to a second person who then throws it back to the first person.

Teacher's Insight

This problem gives students another chance to work within a given set of constraints and to discuss the reasonableness of solutions.

Discussion

Why did Beth's dad have trouble solving the brainteaser?

How do you know whether a solution works?

What is the purpose of this problem?

What is mathematical about this problem?

Problem Set 3, No. 2

Strand: Visual/Spatial Thinking

Focus: Reconfiguration

Task: Using flexibility to solve a problem with multiple conditions and restraints

PROBLEM

2. The figures below are made with toothpicks.

   a. Move exactly 4 toothpicks to make three small squares, all the same size, with no toothpicks left over.

   b. Move exactly 2 toothpicks to make four small squares, all the same size, with no toothpicks left over.

Teacher's Insight

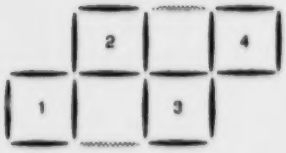
This problem helps students see the value of using manipulatives to help them solve problems. You should have toothpicks available for students to demonstrate their solutions. Students who have difficulty with these problems often find solutions that do not completely meet the conditions. For example, they might use two toothpicks to make a shared side. This problem offers a way to represent "before" and "after" pictures and to represent motion.

ANSWER

Ask the class to share a variety of answers. Answers should include these arrangements:

2. a.

b.



Discussion

What does *exactly* mean?

What method did you use?

Is there another solution?

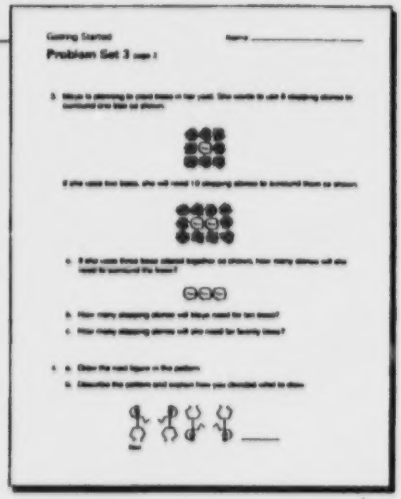
How would a reader know which toothpicks you moved?

How did you represent moving toothpicks?

What made this problem difficult?

Was one problem harder than the other? If yes, why?

How would someone looking at your solution know what you did to solve the problem?



Problem Set 3, No. 3

Strand: Problem Solving

Focus: Numerical patterns

Task: Applying the definition of a pattern and extending the pattern without drawing the next one

PROBLEM

3. Maya is planning to plant trees in her yard.

   She wants to use 8 stepping stones to surround one tree as shown.

   If she uses two trees, she will need 10 stepping stones to surround them as shown.

   a. If she uses three trees placed together as shown, how many stones will she need to surround the trees?

   b. How many stepping stones will Maya need for ten trees?

   c. How many stepping stones will she need for twenty trees?

Teacher's Insight

Searching for patterns is an effective strategy for problem solving. Students sometimes over-generalize about patterns and make wrong assumptions about how to apply them to solving a problem. In this problem students often draw each arrangement of trees and stepping stones up to ten trees and then multiply the number of stones needed for ten trees by 2 to find the number of stones needed for twenty trees.

The arrangement of the trees affects the number of stones. Students may arrange the trees in ways other than one single row, but the pattern may not hold for these arrangements.

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ANSWER

3. a. She will need 12 stepping stones.

   b. Ask the class to share how they decided to arrange the trees. If the trees are placed in a single row, 26 stepping stones will be needed.

   c. If the trees are placed in a single row, 46 stepping stones will be needed.

Discussion

Have students describe what features they looked at to discover a pattern.

Describe this pattern.

What helped you find the pattern?

How can you test the pattern you found?

How does the number of trees affect the number of stepping stones?

If you had arranged the trees a different way (other than in a line), how would that have affected the pattern?

Does your pattern work for any number of trees?

How does finding and using patterns help you solve the problem?

Problem Set 3, No. 4

Strand: Visual/Spatial Thinking and Introduction to Motion

Focus: Visual patterns and using motions in problem solving

Task: Defining a pattern and describing orientation and motion

PROBLEM

4. a. Draw the next figure in the pattern.

   b. Describe the pattern and explain how you decided what to draw.

ANSWER

Ask the class to share a variety of explanations of how they decided on their answer. Answers should include the following figure:

4. a.

   b. The starting figure is flipped, the next one turns the previous one 180°, then the next one flips the previous one. The next figure should therefore turn the previous one 180°.

★HOMEWORK

Problem Set 4

Teacher's Insight

The answer is not unique. What students see as the next figure depends on how they define the pattern. Have students discuss the reasonableness of solutions and ask if they would be able to check their answers.

In describing their patterns, students begin to use more language associated with orientation and motion.

Discussion

How did you decide to draw the figure that came next?

How did you describe the pattern?

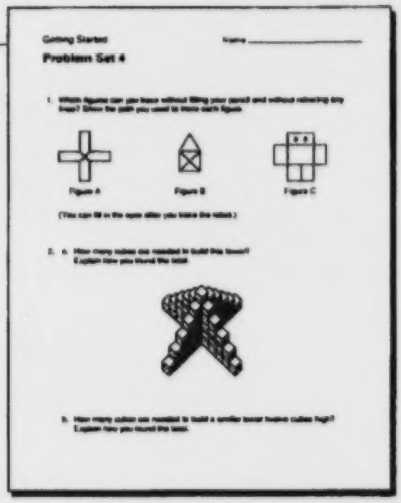
Students may not use mathematical terms to describe reflection.

Could someone have found a different pattern?

How can you check whether a different answer is reasonable?

Why is more than one answer possible?

Justifying Your Answer



Problem Set 4, No. 1

Strand: Visual/Spatial Thinking

Focus: Networks

Task: Using flexibility to find multiple solutions in visual problem solving and to decide that no solution exists

PROBLEM

1. Which figures can you trace without lifting your pencil and without retracing any lines? Show the path you used to trace each figure.

Figure A

Figure B

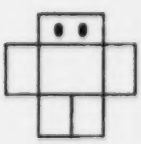


Figure C

(You can fill in the eyes after you trace the robot.)

Teacher's Insight

Because they have had experience with a similar problem, students are more confident approaching this problem. You can focus their attention on explaining how they found their path for tracing the figures and on recording alternate methods. You may be able to highlight some observations about why certain starting points are possible and others are not. However, a lengthy discussion about networks is not appropriate at this time.

Include some discussion, as needed, about how and when students decide that it is not possible to solve a problem.

ANSWER

1. Figures B and C are possible. Figure A is not possible.

Discussion

Could you predict before you started tracing that one figure would be impossible to trace?

How did you make your prediction?

How did you represent the path you took to trace the figures?

Is it possible to use more than one route on any figure?

Why is figure A impossible?

Why should figure A be included in this problem?

How many different starting points are possible for figure B? For figure C?

Do the different starting points have anything in common?

Problem Set 4, No. 2

Strand: Representation and Visual/Spatial Thinking

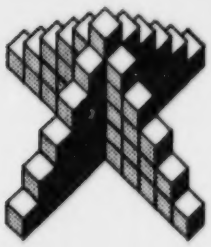
Focus: Interpreting diagrams and visual patterns

Task: Interpreting a diagram and using it to problem solve

PROBLEM

2. a. How many cubes are needed to build this tower?

   Explain how you found the total.



   b. How many cubes are needed to build a similar tower 12 cubes high? Explain how you found the total.

ANSWER

Ask the class to share a variety of explanations of how they decided on their answer.

2. a. 66 cubes

   b. 276 cubes

Teacher's Insight

In this problem, students have to make an assumption about the diagram. Their solution depends on whether they assume that the column of cubes in the middle of the figure is filled or hollow. Students propose and evaluate arguments for why their assumption makes sense.

Students also need to clarify how they interpret the direction "similar tower." Their solutions depend on that interpretation.

Discussion

Did you have any questions about the problem before you tried to solve it?

How did you resolve your questions?

Have students describe and explain solutions, making sure they understand each other's interpretation of the diagram and the directions.

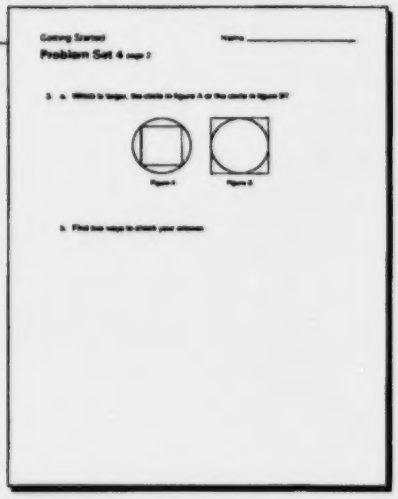
Have them explain why certain interpretations are more productive than others.

If students were not sure how many cubes are in the middle column, ask why that would matter.

How did you interpret "similar tower"?

How did you use your solution for 2.a. to solve 2.b.?

How would you explain to someone who thinks a "similar tower twelve cubes high" takes 132 cubes (66 x 2) that the answer is not correct?



Problem Set 4, No. 3

Strand: Visual/Spatial Thinking and Introduction to Motion

Focus: Testing perceptions and using motions in problem solving

Task: Applying motions to test the congruence of circles

PROBLEM

3. a. Which is larger, the circle in figure A or the circle in figure B?

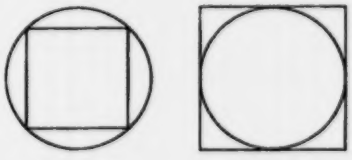


Figure A Figure B

b. Find two ways of checking your answer.

Teacher's Insight

This problem uses an optical illusion to help students realize that they can't always trust what they see. Some will try certain measurements to compare the circles. Students often believe that because measuring yields a number, it is a more valid strategy than one that does not yield a number. The problem gives the class an opportunity to begin to see the value of using motions.

Motions (reflection, translation) are intuitive but powerful strategies for comparing geometric figures. Motions are used in many units in this series. For students who have not studied transformations, motions seem "too easy" to be considered a valid mathematical strategy.

Tracing paper will be useful for demonstrating and practicing the motions informally.

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Answer

3. a. The circles are congruent.

   b. Trace the circle in figure A, then slide the tracing to the right to cover the circle in figure B. Another way is to fold the paper between the two figures so that the figures overlap each other. The circles will match up perfectly.

★Homework

Problem Set 5

Discussion

How did you determine which circle was bigger?

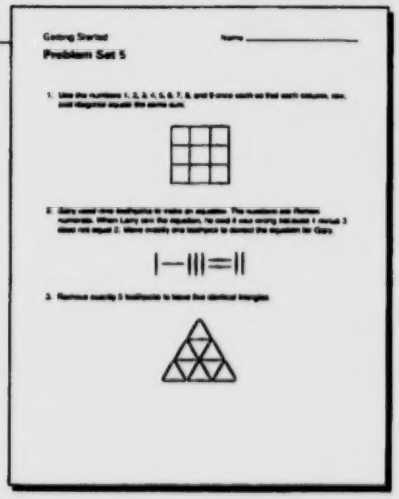
How did you check your decision?

If students describe measuring or used a compass, ask how they decided where to measure. How did you find the center of the circle? How can you be sure you used the diameter? How can you be sure of your accuracy?

Did one of your checks make you feel more certain of your answer than the other check(s) did? Why?

What is mathematical about tracing and matching the circles to compare their size?

Problem Solving and Flexible Thinking



Problem Set 5, No. 1

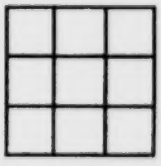
Strand: Problem Solving

Focus: Numerical patterns and testing solutions

Task: Using a diagram to solve a problem with multiple conditions

PROBLEM

1. Use the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9 once each so that each column, row, and diagonal equals the same sum.



ANSWER

1. Ask the class to share a variety of answers. Other answers are 90° rotations of this numerical arrangement.

2	7	6
9	5	1
4	3	8

Teacher's Insight

Students use a variety of strategies to solve this problem. Most use a guess-and-check approach, but some try to use more analytical strategies, such as looking for a relationship between the sum of the counting numbers and the 3 x 3 grid. The answer is unique or not, depending on how one views the effects of rotating or reflecting an arrangement to make a *different* one. This problem foreshadows vocabulary development of *different* in the pentomino problem in Problem Set 6.

Discussion

How did you solve the problem?

What did you have to check to know whether your solution was correct?

Is your answer unique?

Is there anything common to all the solutions?

Are the possible arrangements the same or different? Why?

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Problem Set 5, No. 2

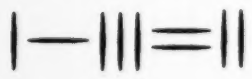
Strand: Visual/Spatial Thinking

Focus: Reconfiguration

Task: Applying flexibility of thinking to reading equations

PROBLEM

2. Gary used nine toothpicks to make an equation. The numbers are Roman numerals. When Larry saw the equation, he said it was wrong because 1 minus 3 does not equal 2. Move exactly one toothpick to correct the equation for Gary.



ANSWER

2. I = III – II (1 = 3 – 2)

Teacher's Insight

Students often read equations as steps for doing an operation rather than as a statement that two expressions are equal. They sometimes think that the equal sign in an equation must precede the answer to an operation. In this problem the solution forces students to redefine an equation and discuss acceptable representations of statements of equality.

Be sure students can represent the numbers 1 through 10 in Roman numerals.

Discussion

What do the Roman numerals stand for?

Does it matter where the equal sign comes?

How do you read the new equation?

What is an equation?

What makes an equation true?

Problem Set 5, No. 3

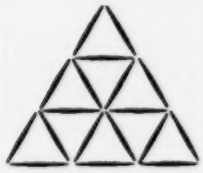
Strand: Visual/Spatial Thinking

Focus: Reconfiguration

Task: Using flexibility in solving visual problems

PROBLEM

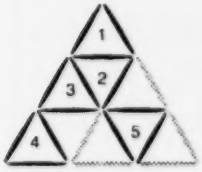
3. Remove exactly 5 toothpicks to leave five identical triangles.



ANSWER

Ask the class to share a variety of answers. Answers should include these formations:

3.



Teacher's Insight

Students need to grasp that *exactly* means no more or no fewer than that number and that *identical* means congruent.

Discussion

How did you solve the problem?

How did you test your answer?

If students talk about the orientation of the triangles, ask if it is still an identical triangle if it faces a different way. Why?

Is there more than one way to solve the problem?

How did you represent your solution strategy?

Is your answer unique?

What makes triangles identical?

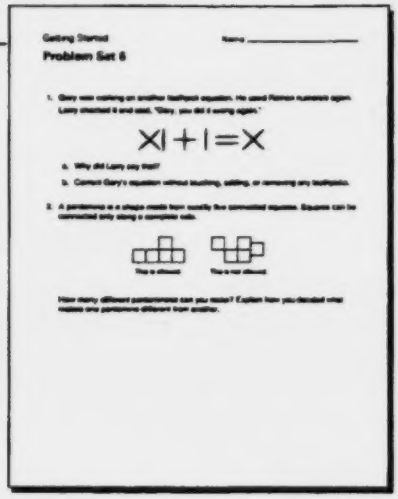
How can we check if the triangles are the same size?

★HOMEWORK

Problem Set 6

Problem Set 6

What Makes It *Different?*



Problem Set 6, No. 1

Strand: Visual/Spatial Thinking

Focus: Reconfiguration

Task: Applying motions and flexibility of thinking to reading equations

PROBLEM

1. Gary was working on another toothpick equation. He used Roman numerals again. Larry checked it and said, "Gary, you did it wrong again."

XI + I = X

   a. Why did Larry say that?

   b. Correct Gary's equation without touching, adding, or removing any toothpicks.

ANSWER

1. a. Larry says this because 11 + 1 is not 10.

   b. Rotate the plane 180° so that the toothpick equation reads X = I + IX (10 = 1 + 9).

Teacher's Insight

This is another problem in which students must read an equation as a statement of an equality. It also gives them a chance to discuss motions in solving problems.

Discussion

If students moved or removed toothpicks, ask them to read the problem again.

What makes this problem different from the problem in Problem Set 5?

What is another toothpick equation that would work the same way?

Problem Set 6, No. 2

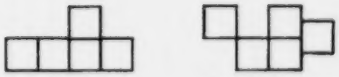
Strand: Visual/Spatial Thinking and Introduction to Motion

Focus: Visual patterns, reconfiguration, and congruence

Task: Defining *different* in a geometric context, advancing the understanding of congruency, using an organized solution strategy

PROBLEM

2. A pentomino is a shape made from *five* connected squares. Squares can be connected only along a complete side. How many *different* pentominoes can you make?



This is allowed. This is not allowed.

Explain how you decided what makes one pentomino different from another.

ANSWER

2. Ask the class to share a variety of answers. There are a total of 12 different arrangements; two are drawn below.

Teacher's Insight

This problem lets students identify a critical process in effective problem solving. It highlights using an organized approach to keep track of possible pentomino arrangements. Students often start drawing arrangements and then have difficulty determining when they have repeated an arrangement.

The problem also highlights the question of what constitutes a *different* figure. Some students estimate that there are many (hundreds) of possible pentominoes. The concept of *what makes something different* is important throughout the study of geometry in this series. Students who have not had experience with the concept of congruence identify a figure as *different* if its orientation is changed. The problem does not address everything about congruence, but the discussion should raise the question and allow students to explore what makes one figure the same as or different from another. The class does not need to reach a consensus at this time.

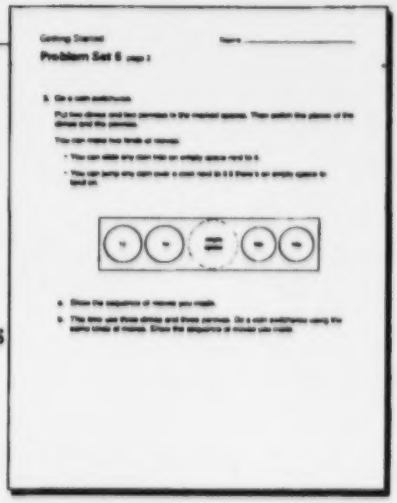
Discussion

How many different pentominoes could you find?

How did you keep track of how many different ones you found?

How did you decide which ones were different?

How did you decide when you had found all the possibilities?



Problem Set 6, No. 3

Strand: Problem Solving, Communication, and Representation

Focus: Numerical patterns, giving/following directions, and representing motion

Task: Using visual thinking and applying experiences with motions and patterns to solve a problem, and representing a series of motions and communicating a series of steps used to complete a task

PROBLEM

3. Do a coin switcheroo.

   Put two dimes and two pennies in the marked spaces. Then switch the places of the dimes and the pennies.

   You can make two kinds of moves:

   - You can slide any coin into an empty space next to it.
   - You can jump any coin over a coin next to it if there's an empty space to land on.

   a. Show the sequence of moves you made.

   b. This time use three dimes and three pennies. Do a coin switcheroo using the same kinds of moves. Show the sequence of moves you made.

Teacher's Insight

In this classic problem you may want to pursue the pattern in moving a successive number of coins from one side of the empty space to the other. Students use a variety of ways to represent the steps they used. Sharing a variety of representations lets students see alternative methods and gives them practice in interpreting representations. Be sure to have coins or chips available for students to demonstrate their solutions to the class.

As needed, discuss the conditions for the moves.

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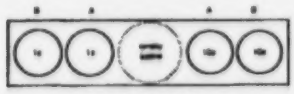
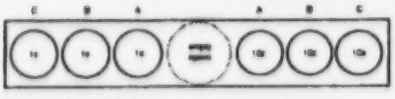
ANSWER

3. a. This switcheroo can be done with as few as eight moves.



1. Slide 10 A
2. Jump 1 A
3. Slide 1 B
4. Jump 10 A
5. Jump 10 B
6. Slide 1 A
7. Jump 1 B
8. Slide 10 B

b. This coin switcheroo can be done with as few as 15 moves



1. Slide 10 A
2. Jump 1 A
3. Slide 1 B
4. Jump 1 A
5. Jump 1 B
6. Slide 10 C
7. Jump 1 A
8. Jump 1 B
9. Jump 1 C
10. Slide 10 A
11. Jump 10 B
12. Jump 10 C
13. Slide 1 B
14. Jump 1 C
15. Slide 10 C

Discussion

What method did you use to solve this problem?

How did you record your moves on your homework paper?

What's the fewest moves needed to do the coin switcheroo?

Is there more than one way to get the fewest moves?

Could you predict the fewest moves with four coins on each side?

What is the greatest number of coins on each side for which you could make a prediction?

Reshaping Mathematics
for Understanding
Getting Started

Blackline Masters

Name _____

Lab A

Drawing Activity

In this lab you will work in pairs. You and your partner will take turns giving and following directions to draw a design that matches, as closely as possible, a given design.

1. Each person will have a card with a design on it. Your design is not like your partner's. Do not show your design to anyone.

2. One person will give directions, and the other will follow them. Sit back to back or in some other way so that you can't see each other's face or each other's work.

3. If you are the person giving directions, you must describe your design so clearly that your partner can make a design just like the one on the card. You may give only spoken directions. You may not use hand motions or other signals, and the person drawing may not ask any questions.

4. When you finish, switch roles. The rules for communicating stay the same, but this time the person drawing may ask questions.

Name _____

Lab B

How to Tie a Bow Activity

Exchange your directions for tying a bow with someone in your group. Each of you must *follow the directions exactly as written.* Do not let what you already know interfere with following your partner's directions. Give each other feedback about your directions, using questions like these:

Could you understand and follow the directions?

What was clear? What was not clear?

If the person drew a diagram, was it helpful?

Did the directions work? By following the directions exactly, could you tie a bow?

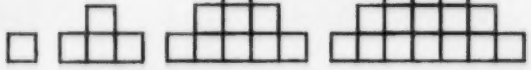
How could you improve the directions?

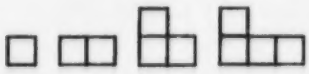
Name _____

Problem Set 1

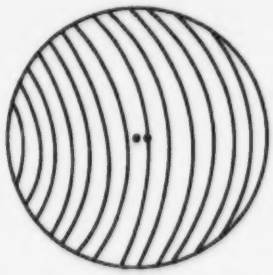
1. Ben turned off his bedroom light and got into bed before the room went dark. His bed is 15 feet from the wall switch. How did Ben do it?

2. Study the arrangements of squares in 2.a. Draw the next three arrangements. Describe your pattern. Do the same for 2.b.

a. 

b. 

3. a. Which dot is at the center of this circle?

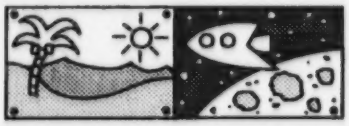


b. Explain your method for deciding.

Name _____

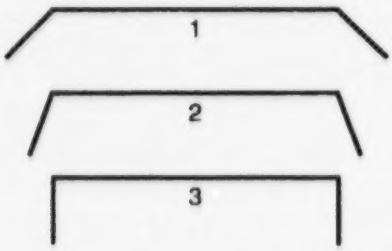
Problem Set 2

1. To hang a picture on the bulletin board, Sara uses 4 thumbtacks, one on each corner. When she overlaps one edge, Sara can hang two pictures using only 6 tacks. If the pictures are all the same size and turned the same way, what is the least number of tacks Sara would need to hang six pictures? Make a diagram to illustrate your answer.



2. Each of the three figures below has a horizontal line segment.

   a. List the segments in order from longest to shortest.



   b. Explain your method for deciding the order.

3. Describe what you see.

Name _____

Problem Set 2 page 2

4. Which figures can you trace without lifting your pencil and without retracing any lines? Show how you traced the figures that were possible.

a.

b.

c.

(You can fill in the eye after you trace the fish.)

Name _____

Problem Set 3

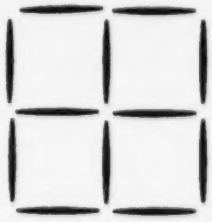
1. Beth made up this brainteaser for her dad to solve:

   How can you throw a ball so it goes a short distance, comes to a dead stop, reverses, and goes the opposite way? You may not bounce the ball off anything, hit it with anything, or tie it to anything.

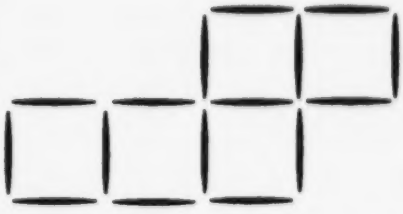
   Beth's dad found this problem difficult to solve. Help him figure out this brainteaser.

2. The figures below are made with toothpicks.

   a. Move exactly 4 toothpicks to make three small squares, all the same size, with no toothpicks left over.



   b. Move exactly 2 toothpicks to make four small squares, all the same size, with no toothpicks left over.



Name _____

Problem Set 3 page 2

3. Maya is planning to plant trees in her yard. She wants to use 8 stepping stones to surround one tree as shown.

If she uses two trees, she will need 10 stepping stones to surround them as shown.

a. If she uses three trees placed together as shown, how many stones will she need to surround the trees?

b. How many stepping stones will Maya need for ten trees?

c. How many stepping stones will she need for twenty trees?

4. a. Draw the next figure in the pattern.

b. Describe the pattern and explain how you decided what to draw.

Name _____

Problem Set 4

1. Which figures can you trace without lifting your pencil and without retracing any lines? Show the path you used to trace each figure.

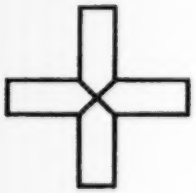


Figure A

Figure B

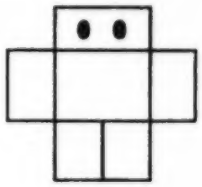
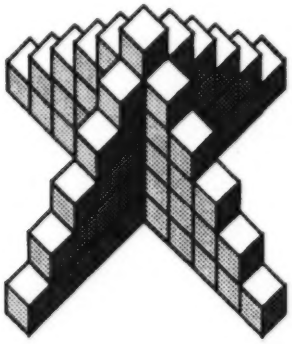


Figure C

(You can fill in the eyes after you trace the robot.)

2. a. How many cubes are needed to build this tower? Explain how you found the total.



b. How many cubes are needed to build a similar tower twelve cubes high? Explain how you found the total.

Name _____

Problem Set 4 page 2

3. a. Which is larger, the circle in figure A or the circle in figure B?

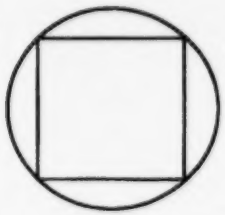


Figure A

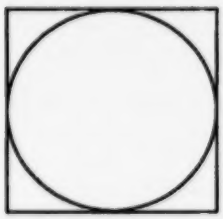


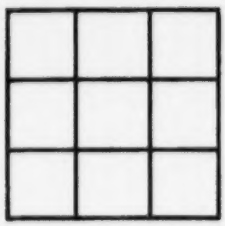
Figure B

b. Find two ways to check your answer.

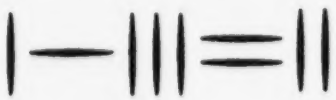
Name _____

Problem Set 5

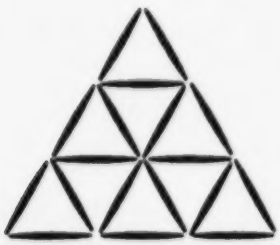
1. Use the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9 once each so that each column, row, and diagonal equals the same sum.



2. Gary used nine toothpicks to make an equation. The numbers are Roman numerals. When Larry saw the equation, he said it was wrong because 1 minus 3 does not equal 2. Move exactly one toothpick to correct the equation for Gary.



3. Remove exactly 5 toothpicks to leave five identical triangles.



Name _____

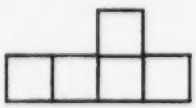
Problem Set 6

1. Gary was working on another toothpick equation. He used Roman numerals again. Larry checked it and said, "Gary, you did it wrong again."

   a. Why did Larry say that?

   b. Correct Gary's equation without touching, adding, or removing any toothpicks.

2. A pentomino is a shape made from exactly five connected squares. Squares can be connected only along a complete side.



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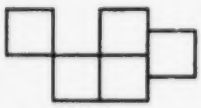


This is not allowed.

   How many *different* pentominoes can you make? Explain how you decided what makes one pentomino different from another.

Name _____

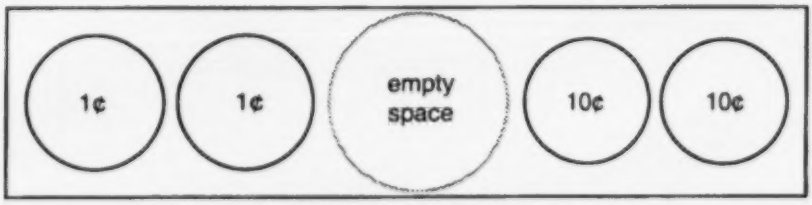
Problem Set 6 page 2

3. Do a coin switcheroo.

   Put two dimes and two pennies in the marked spaces. Then switch the places of the dimes and the pennies.

   You can make two kinds of moves:

   - You can slide any coin into an empty space next to it.
   - You can jump any coin over a coin next to it if there's an empty space to land on.



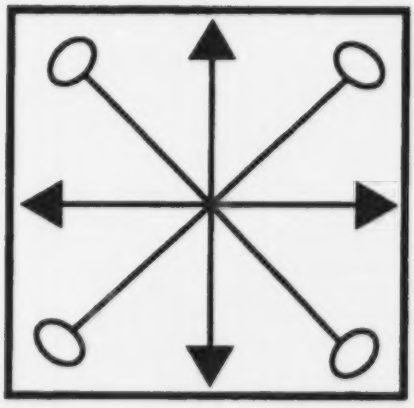
   a. Show the sequence of moves you made.

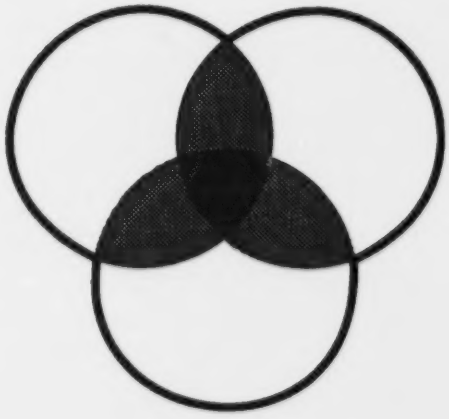
   b. This time use three dimes and three pennies. Do a coin switcheroo using the same kinds of moves. Show the sequence of moves you made.

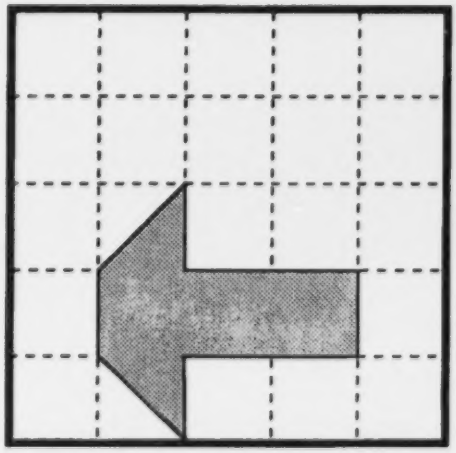
Reshaping Mathematics for Understanding Getting Started

Resources

DRAWING ACTIVITY CARDS







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